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Theoretical Analysis for Liquid Drop Model Parameter Estimation

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ABSTRACT

Nuclear incompressibility is an important parameter that depicts the equation of state of nuclear matter and plays a central role in understanding nuclear matter properties. In this work, a theoretical analysis for the estimation of liquid drop model coefficients using a combination of M3Y-type interaction and the semi empirical mass formula for the liquid drop model is presented. The nuclear incompressibility obtained through estimation of other key parameters like saturation density, Fermi momentum and Coulomb constant predict nuclear matter properties which agree well with those from other methods and as such show the uniqueness of the current method.

Keywords Nuclear incompressibility, Nuclear Saturation Density and Coulomb constant

INTRODUCTION

The liquid drop model (LDM) in nuclear physics treats the nucleus as a drop of incompressible nuclear fluid. It was first proposed by George Gamow and then developed by Neils Bohr and John Archibald Wheeler. The nucleus is made of nucleons (protons and neutrons), which are held together by the nuclear force. This is very similar to the structure of spherical liquid drop made of microscopic molecules. This crude model does not explain all the properties of the nucleus, but does explain the spherical shape of most nuclei. The liquid drop model is based on the saturation of the nuclear forces one relates the energy of the system to its geometric properties. It also helps to predict the nuclear binding energy and to assess how much is available for consumption [1-3].

The basic assumption of the LDM is that the nucleus is a charged, nonpolar liquid drop held together by the nuclear force. In the simplest case, would be a droplet of composed of nonpolar molecules. For such system, the following properties are observed: The attractive force is short range; that is, there is a relatively sharp boundary at the surface. The force is saturated; that is, all nucleons in the bulk of the liquid are bound equally, independent of radius. The nucleus is incompressible in its ground state, which accounts for the nearly uniform density distribution and constant average binding energy. Surface tension is created by the loss in binding for nucleons on the nuclear surface, an effect that leads to a spherical shape to minimize the surface energy [4]. Additionally, to gain information about nuclear equation of state (EOS) requires modeling interactions among nucleons in nucleon-nucleon (NN) scattering and this forms the basis of the Bethe- Weizsäcker formula or simply, semi empirical mass formula [5], this formula given by equation(1) allows the estimation of nuclear incompressibility(K_o), an empirical property used in characterizing EOS.

$$B(A,Z) = a_v A - a_s A^{\frac{2}{3}} - \frac{a_c Z(Z-1)}{A^{\frac{1}{3}}} - \frac{a_{sym}(A-2Z)^2}{A} \pm \delta(A,Z) \quad (1)$$

Where a_v , a_s , a_c, a_{sym} and δ are volume term, surface term, Coulomb term, symmetry term and pairing term respectively. The coefficients a's are fitted parameters.

Evidently, the fact that NM has a constant density points towards its incompressibility and forms the centre point of the theoretical analysis in this work. To this end, this study aims to determine theoretically, parameters of LDM for investigating nuclear matter properties.

THEORETICAL BACKGROUND

The M3Y-Density-Dependent interaction

A finite range effective NN interaction, successfully and widely used in double folding calculations of the heavy-Ion potential, is the so-called MY3-interaction. This interaction has been derived by fitting its matrix elements in an oscillator basis (the oscillator parameters were chosen to produce the ground state of $^{16}_{8}O$) to the elements of the G-matrix obtained with the Reid soft-core NN interaction. In order to study EOS of the cold nuclear matter using the M3Y-interaction, a realistic density dependence was introduced in order to describe the equilibrium density and nuclear incompressibility [5].

For NN interactions, the popular choices have frequently been based on the M3Y interactions which were designed to reproduce the G-matrix elements of Reid and Paris NN interactions in an oscillator basis. These are M3Y-Reid and M3Y-Paris and Reid-Elliott interactions respectively. Explicitly, the direct parts including the following;

M3Y-Reid-Elliot [5], [6-14].

$$V_D\text{(r)}=[7999\frac{\exp(-4r)}{4r}-2134\frac{\exp(-2.5r)}{2.5r}]MeV \qquad (2)$$

M3Y-Paris, [5], [9-11].

$$V_D(r)=[11062\frac{\exp(-4r)}{4r}-2538\frac{\exp(-2.5r)}{2.5r}]MeV \qquad (3)$$

and,

M3Y-Reid[5,9].

$$V_D\text{(r)}=[14743.375\frac{\exp(-4r)}{4r}-3296.563\frac{\exp(-2.5r)}{2.5r}]MeV \qquad (4)$$

The exchange parts in the finite-range exchanges are:

M3Y-Reid-Elliott

$$V_{EX(r)}=[4631.38\frac{\exp(-4r)}{4r}-1787.13\frac{\exp(-2.5r)}{2.5r}-7.8474\frac{\exp(-0.7072r)}{0.7072r}]MeV \qquad (5)$$

M3Y-Paris:

$$V_{EX(r)}=[-1524.25\frac{\exp(-4r)}{4r}-518.75\frac{\exp(-2.5r)}{2.5r}-7.8474\frac{\exp(-0.7072r)}{0.7072r}]MeV \qquad (6)$$

and,

M3Y-Reid:

$$V_{EX}(r)=[-2312.625\frac{\exp(-4r)}{4r}-264.813\frac{\exp(-2.5r)}{2.5r}-7.864\frac{\exp(-0.7072r)}{0.7072r}]MeV \qquad (7)$$

Where

$V_D(r)$, $V_{EX}(r)$and $r$represent direct parts, exchange parts and the interaction distance respectively.

A realistic description of NM properties can be obtained by introducing an appropriate density dependence in the original M3Y-interaction by multiplying the original interaction with a density dependent (DD) factor $g\ (\rho,\epsilon)$.When this is done, interactions (2),(3),(4),(5),(6) and (7) hereafter becomes DDM3Y Reid-Elliott,DDM3Y Reid and DDM3Y Paris respectively.

Density Dependent Effective Interaction

The general expression for the density dependent effective NN interaction potential $V(r)$ is written as [5], [6,7, 15, 16].

$$V(r,\rho,\epsilon) = t^{M3Y}(r,\epsilon)g(\rho,\epsilon) \tag{8}$$

where the realistic effective M3Y interaction potential supplemented by zero range pseudopotential $t^{M3Y}$is given by[5, 9].

$$t_{RE}^{M3Y}(r,\epsilon) = 7999\frac{\exp(-4r)}{4r} - 2134\frac{\exp(-2.5r)}{2.5r} + J_{oo}(\epsilon)\delta(r) \tag{9}$$

$$t_{P}^{M3Y}(r,\epsilon) = 11062\frac{\exp(-4r)}{4r} - 2538\frac{\exp(-2.5r)}{2.5r} + J_{oo}(\epsilon)\delta(r) \tag{10}$$

and

$$t_{R}^{M3Y}(r,\epsilon) = 14943\frac{ext(-4r)}{4r} - 3296.563\frac{\exp(-2.5r)}{2.5r} + J_{OO}(\epsilon)\delta(r) \tag{11}$$

where the constants 4 and 2.5 have the dimensions offm^{-1}, δ(r) has the dimension of fm^{-3}, t_{RE}^{M3Y}, t_{P}^{M3Y} and t_{R}^{M3Y} and are the direct parts of Reid-Elliott,Paris and Reid interactions respectively.

The zero-range pseudopotential

The zero-range pseudopotential $J_{OO}(\epsilon)$ represents the single-nucleon exchange term and is given by [5], [6-8], [12,16].

$$J_{OO}(\epsilon) = -276(1-\alpha\epsilon)[MeV fm^{3}] \tag{12}$$

Where $\alpha = 0.005MeV^{-1}$ is the energy dependence parameter and $\epsilon$is the saturation energy per nucleon.

The Density Dependent Parameters

The density dependent part is given is general form as [5], [6,7, 15,16]:

$$g(\rho,\epsilon) = C(1-\beta(\epsilon)\rho^{n}) \tag{13}$$

which takes care of the higher order exchange effects and the Pauli blocking effects. This density dependence changes sign at high densities which is of crucial importance in fulfilling the saturation condition as well as given different K_o values with different values of n for the nuclear EOS. The value of the parameter $n = \frac{2}{3}$ was originally taken by Meyers worked well in both in the single and in double folding calculations with the factorized density dependence for the higher energy heavy ion scattering and the cluster radioactivity [6]. The other two parameters $C$and $\beta(\epsilon)$, has been fitted to reproduce the saturation conditions. The saturation condition for cold nuclear matter is not fulfilled here due to attractive character of the M3Y forces. However, the realistic description of nuclear matter properties can be obtained with this density dependent M3Y effective interaction. The density dependence parameters are obtained by reproducing the saturation energy per nucleon and the saturation nucleonic density of the spin and isospin symmetric cold infinite nuclear matter.

The Energy per Nucleon

The energy per nucleon ϵ obtained using the effective NN interaction $V(r)$ for the spin and isospin symmetric cold infinite nuclear matter is given by [6-8]:

$$\epsilon = \frac{3\hbar^2 k_F^2}{10m} + g\frac{(\rho,\epsilon)\rho J_v}{2} \tag{14}$$

Where m is the nucleonic mass which is equal to 931.4943 $\frac{MeV}{C^2}$ and J_v represents the volume integral of the M3Y interaction supplemented by the zero-range psuedopotential having the form

$$J_v(\epsilon) = \iiint t^{M3Y}(r,\epsilon)d^3r = 7999\left(\frac{4\pi}{3^3}\right) - 2134\left(\frac{4\pi}{2.5^3}\right) + J_{oo}(\epsilon) \tag{15}$$

Equation (14) can be written in terms of equation (13) as

$$\epsilon = \frac{3\hbar^2 k_F^2}{10m} + \rho J_V \mathrm{C}\frac{(1-\beta\rho^{\frac{2}{3}})}{2} \tag{16}$$

The equilibrium density of the spin and isospin symmetric cold infinite nuclear matter is determined from the saturation condition $\frac{\partial\epsilon}{\partial\rho} = 0$ from equation (16) which yields the equation

$$\frac{\partial\epsilon}{\partial\rho} = \frac{\partial}{\partial\rho}\left(\frac{3\hbar^2 k_F^2}{10m} + \rho J_V \mathrm{C}\frac{(1-\beta\rho^{\frac{2}{3}})}{2}\right) \tag{17}$$

$$= \frac{\partial}{\partial\rho}\left(\frac{3\hbar^2 k_F^2}{10m}\right) + \frac{\partial}{\partial\rho}\left(\rho J_V \mathrm{C}\frac{(1-\beta\rho^{\frac{2}{3}})}{2}\right)$$

now,

$$\frac{\partial}{\partial\rho}(\frac{3\hbar^2k_F^2}{10m})=\frac{3\hbar^2}{10m}\frac{\partial(1.5\pi^2\rho)^{\frac{2}{3}}}{\partial\rho}$$

$$=\frac{3\hbar^2}{10m}((1.5\pi^2)^{\frac{2}{3}}\frac{\partial}{\partial\rho}\rho^{\frac{2}{3}}$$

$$=\frac{3\hbar^2}{10m}(1.5\pi^2)^{\frac{2}{3}}\frac{2}{3}\rho^{\frac{-1}{3}}$$

$$=\frac{\hbar^2}{5m}(1.5\pi^2)^{\frac{2}{3}}\rho^{\frac{-1}{3}}$$

$$=\frac{\hbar^2}{5m}(1.5\pi^2)^{\frac{2}{3}}\rho^{\frac{2}{3}-1}$$

$$=\frac{\hbar^2}{5m}(1.5\pi^2)^{\frac{2}{3}}\rho^{\frac{2}{3}}\rho^{-1}$$

$$=\frac{\hbar^2}{5m}\frac{(1.5\pi^2\rho)^{\frac{2}{3}}}{\rho}$$

$$=\frac{\hbar^2k_F^2}{5m\rho} \tag{18}$$

similarly,

$$\frac{\partial}{\partial\rho}[\rho J_V C\frac{(1-\beta\rho^{\frac{2}{3}})}{2}]=\frac{\partial}{\partial\rho}[\frac{J_V C(\rho-\beta\rho^{\frac{5}{3}})}{2}]$$

$$=J_V C[\frac{\left(1-\frac{5}{3}\rho\beta(\epsilon)\rho^{\frac{2}{3}}\right)}{2}] \tag{19}$$

Substitution of equations (19) and (18) into (17) yields

$$\frac{\partial\epsilon}{\partial\rho}=[\frac{\hbar^2k_F^2}{5m\rho}+J_V C(\frac{\left(1-\frac{5}{3}\rho\beta(\epsilon)\rho^{\frac{2}{3}}\right)}{2}]=0 \tag{20}$$

Equations (16) and (20) with the saturation condition can be solved simultaneously for fixed values of the saturation energy per nucleon and the saturation density of the spin and isospin symmetric cold infinite nuclear matter to obtain the values of density dependent parameters $\beta(\epsilon)$ and C. The density dependent parameter C is obtained from equation (20) as:

$$\frac{2\hbar^2k_F^2}{5\rho m}+J_V C\left(1-\frac{5}{3}\beta(\epsilon)\rho^{\frac{2}{3}}\right)=0$$

$$J_V C\left(1-\frac{5}{3}\beta(\epsilon)\rho^{\frac{2}{3}}\right)=\frac{-2\hbar^2k_F^2}{5m\rho}$$

$$C\left(1-\frac{5}{3}\beta(\epsilon)\rho^{\frac{2}{3}}\right)=\frac{-2\hbar^2k_F^2}{5m\rho J_V}$$

$$C = \frac{-2\hbar^2 k_F^2}{5m\rho J_V\left(1-\frac{5}{3}\beta(\epsilon)\rho^{\frac{2}{3}}\right)} \quad (21)$$

Now, substitutes equation (21) into equation (16) and solve for $\beta(\epsilon)$

$$\epsilon = [\frac{3\hbar^2 k_F^2}{10m} + \rho J_V C \frac{\left(1-\frac{5}{3}\rho\beta(\epsilon)\rho^{\frac{2}{3}}\right)}{2}]$$

$$\epsilon = \frac{3\hbar^2 k_F^2}{10m} + \frac{\rho J_V}{2} C[\frac{-2\hbar^2 k_F^2}{5m\rho J_V\ (1-\frac{5}{3}\beta(\epsilon)\rho^{\frac{2}{3}})}]\left(1-\beta(\epsilon)\rho^{\frac{2}{3}}\right)$$

$$= \frac{3\hbar^2 k_F^2}{10m} - \rho J_V\left(\frac{-2\hbar^2 k_F^2}{10m\left(3-5\beta(\epsilon)\rho^{\frac{2}{3}}\right)}\right)\left(1-\beta(\epsilon)\rho^{\frac{2}{3}}\right)$$

Simplifying and make $\beta(\epsilon)$ the subject formula

$$10m\epsilon\left(3-5\beta(\epsilon)\rho^{\frac{2}{3}}\right) = 3\hbar^2 k_F^2\left(3-5\beta(\epsilon)\rho^{\frac{2}{3}}\right) - 6\hbar^2 k_F^2\left(1-\beta(\epsilon)\rho^{\frac{2}{3}}\right)$$

$$30m\epsilon - 50m\epsilon\beta(\epsilon)\rho^{\frac{2}{3}} = 9\hbar^2 k_F^2 - 15\hbar^2 k_F^2\beta(\epsilon)\rho^{\frac{2}{3}} - 6\hbar^2 k_F^2 + 6\hbar^2 k_F^2\beta(\epsilon)\rho^{\frac{2}{3}}$$

$$30m\epsilon - 50m\epsilon\beta(\epsilon)\rho^{\frac{2}{3}} = 3\hbar^2 k_F^2 - 9\hbar^2 k_F^2\beta(\epsilon)\rho^{\frac{2}{3}}$$

$$30m\epsilon - 3\hbar^2 k_F^2 = 50m\epsilon\beta(\epsilon)\rho^{\frac{2}{3}} - 9\hbar^2 k_F^2\beta(\epsilon)\rho^{\frac{2}{3}}$$

$$30m\epsilon - 3\hbar^2 k_F^2 = \beta(\epsilon)\rho^{\frac{2}{3}}(50m\epsilon - 9\hbar^2 k_F^2)$$

$$\beta(\epsilon) = \frac{30m\epsilon - 3\hbar^2 k_F^2}{\rho^{\frac{2}{3}}(50m\epsilon - 9\hbar^2 k_F^2)} \quad (22)$$

Divide both numerator and denominator of equation (22) by $\hbar^2 k_F^2$ to get

$$\beta(\epsilon) = \frac{\frac{-30m\epsilon}{\hbar^2 k_F^2}+3}{\rho^{\frac{2}{3}}(\frac{-50m\epsilon}{\hbar^2 k_F^2}+9)}$$

$$= \frac{3-\frac{3(10m\epsilon)}{\hbar^2 k_F^2}}{\rho^{\frac{2}{3}}[9-\frac{5(10m\epsilon)}{\hbar^2 k_F^2}]}$$

$$= \frac{3-3p}{\rho^{\frac{2}{3}}(9-5p)} \quad (23)$$

where,

$$p = \frac{10m\epsilon}{\hbar^2 k_F^2} = \frac{10m\epsilon}{\hbar^2(1.5\pi^2\rho)^{\frac{2}{3}}} \quad (24)$$

$$\beta(\epsilon) = [\frac{\frac{(3-3p)}{(9-5p)}}{\rho^{\frac{2}{3}}}] \tag{25}$$

ESTIMATION METHOD

Coulomb constant term

The third term of the semi-empirical mass formula given by equation (1), $-a_c Z(Z-1)A^{\frac{-1}{3}}$, is derived from the Coulomb interaction among protons, and is proportional to Z. The electric repulsion between each pair of protons in the nucleus contributes toward decreasing its binding energy. The nucleus can be considered as a sphere of uniform charge density to a very rough approximation.

The potential energy of such a charge distribution can be shown to be [1].

$$E = \frac{3}{5}(\frac{1}{4\pi\varepsilon_o})\frac{Q^2}{R} \tag{26}$$

Where Q is the total charge and R is the radius of the sphere. Identifying Q with Ze, and noting as above that the radius is proportional to $A^{\frac{1}{3}}$, we get close to the Coulomb term (a_c). However, because electrostatic repulsion will only exist for more than one proton, Z^2 becomes $Z(Z-1)$.

The value of $a_c$can be approximately calculated as follows:

The value of a_c can be approximately calculated using the potential energy due to charge distribution given by

$$E = \frac{1}{4\pi\varepsilon_o}\int dq\,(\underset{r}{\rightarrow})\frac{q(\underset{r}{\rightarrow})}{|\vec{r}|} = \frac{1}{4\pi\varepsilon_o}\int d^3\vec{r}\,\frac{\rho q(\underset{r}{\rightarrow})}{|\vec{r}|} \tag{27}$$

Where the charge $q(r)$ is expressed in terms of charge density ρ as $q(r) = \frac{4}{3}\pi r^3\rho = Q(\frac{r}{R})^3$. If R and Q are expressed respectively as

$R \approx r_o A^{\frac{1}{3}}$and $Q = Ze$, equation (27) may be simplified as follows:

$$E = \frac{1}{4\pi\varepsilon_o}\int_0^R dr\,4\pi r^2\rho\frac{q(r)}{r}$$

$$= \frac{1}{4\pi\varepsilon_o}[4\pi\int_O^R dr\,\frac{3Q}{4\pi R^3}r^2\,Q(\frac{r}{R})^3\frac{1}{r}]$$

$$= \frac{1}{4\pi\varepsilon_o}\int_0^R dr\,\frac{3Q^2r^4}{R^6}$$

$$= \frac{1}{4\pi\varepsilon_o}\frac{3}{5}\frac{Q^2}{R}$$

$$= \frac{3}{5}\left(\frac{1}{4\pi\varepsilon_o}\right)\frac{(Ze)^2}{r_o A^{\frac{1}{3}}}$$

$$= \frac{3}{5}\left(\frac{1}{4\pi\varepsilon_o}\right)\frac{Z(Z-1)}{r_o A^{\frac{1}{3}}}$$

$$= a_c \frac{Z(Z-1)}{A^{\frac{1}{3}}} \tag{28}$$

Where

$$a_c = \frac{3e^2}{5r_o}\left(\frac{1}{4\pi\,\varepsilon_o}\right) \tag{29}$$

In equation (27), the quantity of interest is the Coulomb radius constant r_o, and is expressed as

$$r_o = \frac{3e^2}{5a_c}\left(\frac{1}{4\pi\varepsilon_o}\right) \tag{30}$$

One of the most directly derivable information provided by the mass formula which is the magnitude of the Coulomb radius constant r_o contained within the expression for the Coulomb energy constant a_c assuming a uniform volumedistribution of charge [1,2,7,15,17].

Nuclear incompressibility

The nuclear incompressibility (K_o) or the compression modulus of the spin and isospin symmetric cold infinite NM is defined as a measure of the curvature of an EOS at saturation density and measures the stiffness of EOS. It is given by [5,7,8,16].

$$K_o = K_F^2 \frac{\partial^2 \epsilon}{\partial K_F^2} = 9\rho^2 \frac{\partial^2 \epsilon}{\partial \rho^2}\Big|_{\rho=\rho_o} \tag{31}$$

The Fermi momentum k_F for the spin and isospin symmetric infinite nuclear matter is defined by

$$K_F = (1.5\pi^2\rho)^{\frac{1}{3}} \tag{32}$$

where $\rho$is the nucleonic density while$\rho_o$is the saturation density for the spin and isospin symmetric cold infinite nuclear matter. The effective NN interaction $V(r)$ is assumed to be density dependent and energy dependent and hence becomes functions of both density and energy. Once the density dependence parameters $\beta(\epsilon)$ and C have been fixed for a saturation energy per nucleon and a saturation density of a spin and isospin symmetric cold

INM, the incompressibility K_o can be evaluated from the equation given by equation (31).

Substitution of equation (20) into equation (31) yields

$$K_o = 9\rho^2 \frac{\partial}{\partial\rho}[\frac{\hbar^2 K_F^2}{5m\rho} + \frac{J_V C}{2}(1 - \frac{5}{3}\beta(\epsilon)\rho^{\frac{2}{3}})]$$

$$= 9\rho^2 \frac{\partial}{\partial\rho}(\frac{\hbar^2 K_F^2}{5m\rho} + \frac{\rho J_V C}{2} - \frac{5\rho J_V C\beta(\epsilon)}{2}\frac{\rho^{\frac{2}{3}}}{3})$$

$$= 9\rho^2[\frac{\partial}{\partial\rho}(\frac{\hbar^2 K_F^2}{5m\rho}) - (\frac{5\rho J_V C\beta(\epsilon)}{2}\frac{\rho^{\frac{2}{3}}}{3})]$$

$$= 9\rho^2[\frac{\partial}{\partial\rho}\frac{\hbar^2(1.5\pi^2\rho)^{\frac{2}{3}}\rho^{-1}}{5m} - \frac{\partial}{\partial\rho}(\frac{5\rho J_V C\beta(\epsilon)}{2}\frac{\rho^{\frac{2}{3}}}{3})]$$

$$= 9\rho^2[\frac{\partial}{\partial\rho}\frac{\hbar^2(1.5\pi^2)^{\frac{2}{3}}\rho^{\frac{2}{3}}\rho^{-1}}{5m} - \frac{\partial}{\partial\rho}(\frac{5\rho J_V C\beta(\epsilon)}{2}\frac{\rho^{\frac{2}{3}}}{3})]$$

$$= 9\rho^2[\frac{\partial}{\partial\rho}\frac{\hbar^2(1.5\pi^2)^{\frac{2}{3}}\rho^{\frac{-1}{3}}}{5m} - \frac{2}{3}.\frac{5\rho J_V C\beta(\epsilon)}{2}.\frac{\rho^{\frac{-1}{3}}}{3})]$$

$$= 9\rho^2[\frac{-1}{3}.\frac{\hbar^2(1.5\pi^2)^{\frac{2}{3}}\rho^{\frac{-4}{3}}}{5m} - \frac{5\rho J_V C\beta(\epsilon)\rho^{\frac{-1}{3}}}{9}]$$

$$= 9\rho^2[\frac{-1}{3}.\frac{\hbar^2(1.5\pi^2)^{\frac{2}{3}}.\rho^{\frac{2}{3}}.\rho^{\frac{-6}{3}}}{5m} - \frac{5\rho J_V C\beta(\epsilon)\rho^{\frac{-1}{3}}}{9}]$$

$$= 9\rho^2[\frac{-1}{3}.\frac{\hbar^2(1.5\pi^2\rho)^{\frac{2}{3}}.\rho^{-2}}{5m} - \frac{5\rho J_V C\beta(\epsilon)\rho^{\frac{-1}{3}}}{9}]$$

$$= 9\rho^2[\frac{-1}{3}.\frac{\hbar^2 K_F^2.\rho^{-2}}{5m} - \frac{5\rho J_V C\beta(\epsilon)\rho^{\frac{-1}{3}}}{9}]$$

$$K_o = [\frac{-3\hbar^2 K_F^2}{5m} - 5J_V C\beta(\epsilon)\rho^{\frac{5}{3}}]\Big|_{\rho=\rho_o} \quad (33)$$

Which was theoretically derived using equations (31), (32) and (20) with J_V , C, and $\beta(\epsilon)$ given by equations (16), (21) and(25) respectively. Hence, through equations (30), (32 and, (33), we obtained key expressions important NM properties studied in this work. The proceeding section gives the results following these derivations.

RESULTS AND DISCUSSION

The Coulomb constant radius r_o is calculated from equation (30) as a function of a_c with already established value and used to determine ρ_o according to equation (30). The value of

ρ_o obtained was used to determine k_{F_o} using equation (32), which was in turn used to obtain K_o according to equations (33).

The results obtained for these parameters are presented in Tables 1-2 alongside the values for already established parameters such as a_c which was used to obtain r_o and this for $\rho_o = 0.1 fm^{-3}$. The energy dependence parameter α fixed at 0.1 MeV^{-1} and the saturation energy per nucleon $\epsilon_o = -16MeV$.

Table 1. Calculated r_o at established values of $\alpha = 0.1\ MeV^{-1}$, $\epsilon_o = -16MeV$ and a_c.

S/N	$\epsilon_o[MeV]$	$a_c[MeV]$	$r_o[fm]$
1.	15.260	0.689	1.254
2.	15.777	0.710	1.217
3.	15.409	0.695	1.243
4.	15.777	0.710	1.217
5.	15.850	0.714	1.208

Table 2: Calculated K_o at different ρ_o based on r_o obtained

S/N	$\rho_o[fm^{-3}]$	$\beta[fm^{-2}]$	C	$K_o[MeV]$
1.	0.1211	2.0049	2.4724	295.5501
2.	0.1325	1.8730	2.3167	298.9501
3.	0.1243	1.9659	2.4262	296.4862
4.	0.1325	1.8730	2.3167	298.9507
5.	0.1353	1.8435	2.2820	299.7840

As can be inferred from Table 2, the current theoretical analysis gave $\boldsymbol{K_o}$ to be 297.9442 $\boldsymbol{MeV}$ which agrees well with results from other methods such as the infinite nuclear model and experimental data with $\boldsymbol{K_o = 300 \pm 25MeV}$.

CONCLUSION

The present theoretical estimate of nuclear incompressibility is in reasonably close with other theoretical estimates obtained by INM model, using the Seyler-Blanchard interaction established in [7] and as such, these findings justify the uniqueness of the current method.

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